

# DISEÑO ASISTIDO POR ORDENADOR

4º Curso Ingeniería Informática

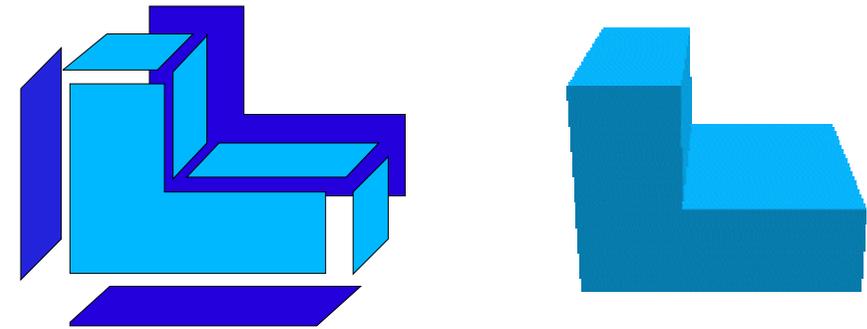
Dpto. Lenguajes y Sistemas Informáticos

Curso 2005/06

## Tema 5: Modelado de sólidos (3ª Parte)

J.C. Torres, P. Cano

### 5.3. Modelo de fronteras.

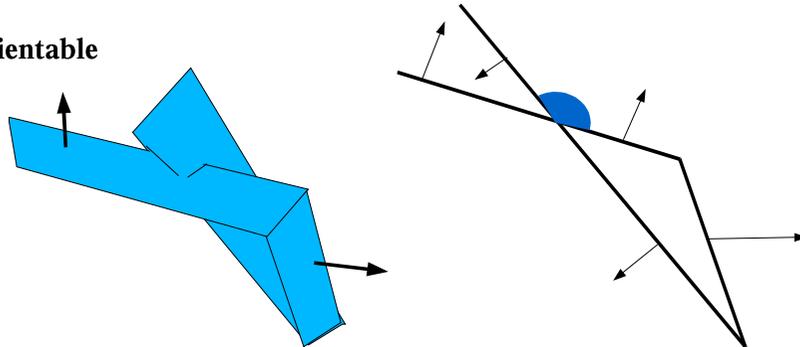


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### 5.3. Modelo de fronteras: Modelo algebraico

Volumen encerrado por una superficie:

- Cerrada
- Orientable



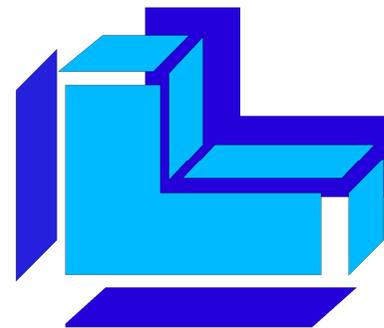
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### 5.3. Modelo de fronteras.

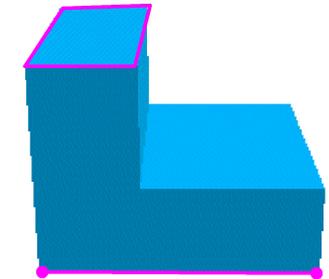
Geometría

+

Topología



Las caras están delimitadas por aristas



Las aristas están delimitadas por dos vértices.

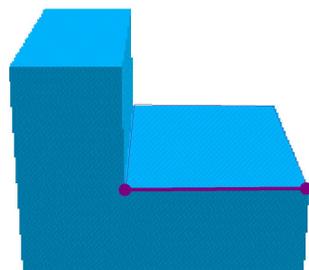
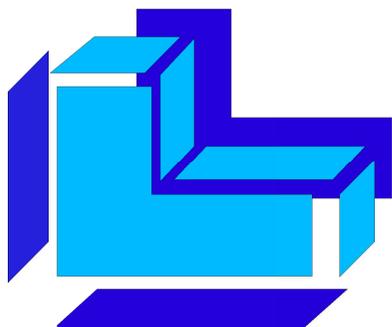
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### 5.3. Modelo de fronteras.

Geometría

+

Topología



Cada arista es compartida por dos caras con la misma orientación.

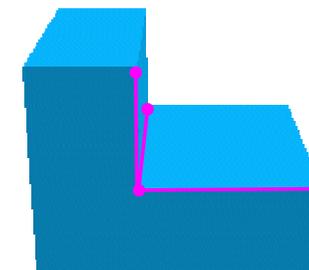
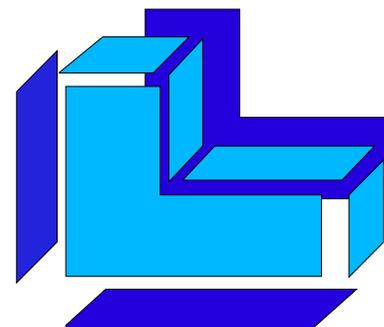
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### 5.3. Modelo de fronteras.

Geometría

+

Topología

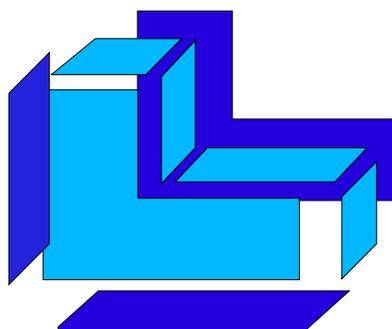


En cada vértice confluyen al menos tres aristas

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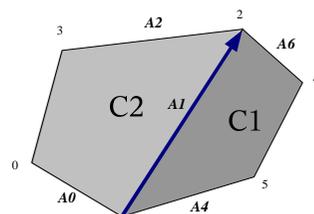
### 5.3. Modelo de fronteras: Representación

Solo geometría: Lista de caras



Sólido = lista de caras  
 Cara = lista de vértices

| Cara | Vértices                               |
|------|--|
| 1    | (0,0,0) (0,2,-1) (0,3,2) (0,1,2) NULL  |
| 2    | (0,2,-1) (2,3,-1) (2,4,1) (0,3,2) NULL |
| 3    |  |



¿Cómo comprobar que cada arista está compartida por dos caras con la misma orientación?

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### 5.3. Modelo de fronteras: Representación

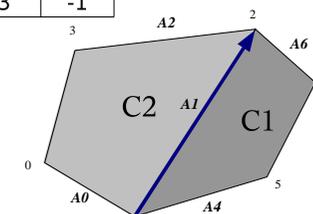
Caras - aristas - vértices

Sólido = lista de caras  
 Cara = lista de aristas  
 Arista = par de vértices

| Cara | Aristas |   |   |   |      |
|------|---------|---|---|---|------|
| 1    | 0       | 1 | 2 | 3 | NULL |
| 2    | 4       | 5 | 6 | 1 | NULL |
| 3    |         |   |   |   |      |

| Aristas | V <sub>i</sub> | V <sub>f</sub> | C <sub>izq</sub> | C <sub>der</sub> |
|---------|----------------|----------------|------------------|------------------|
| 0       | 0              | 1              | 1                | 3                |
| 1       | 1              | 2              | 1                | 2                |
| 2       | 2              | 3              | 1                | 4                |
| 3       | 3              | 0              | 1                | 5                |
| 4       | 1              | 5              | 2                | 3                |
| 5       | 5              | 4              | 2                | 6                |
| 6       | 4              | 2              | 2                | 4                |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



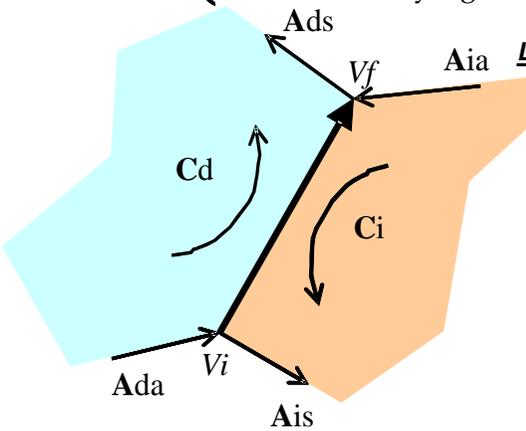
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### 5.3. Modelo de fronteras: Representación

#### Aristas con alas

Sólido = lista de aristas

Arista =  $\begin{cases} 2 \text{ vértices} \\ 2 \text{ caras (una que recorre la arista derecha y otra inversa)} \\ \text{arista anterior y siguiente al recorrer cada cara} \end{cases}$



**Leyenda:**

- Vi - Vértice inicial de la arista
- Vf - Vértice final de la arista
- Cd - Cara que recorre la arista al derecho
- Ci - Cara que recorre la arista invertida
- Ada - Arista anterior según Cd
- Ads - Arista siguiente según Cd
- Aia - Arista anterior según Ci
- Ais - Arista siguiente según Ci

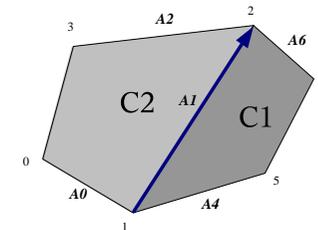
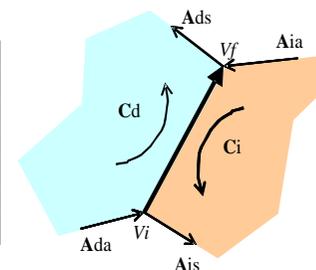
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 1  |    | 3   | 1   |     |     |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 1  |    | 1   | 3   |     |     |
| 3       | 3  | 0  | 1  |    | 2   | 0   |     |     |
| 4       | 5  | 1  |    | 2  |     |     | 1   | 5   |
| 5       | 4  | 5  |    | 2  |     |     | 4   | 6   |
| 6       | 2  | 4  |    | 2  |     |     | 5   | 1   |

| Cara | Arista | Normal  |
|------|--------|---------|
| 1    | 4      | (1,0,0) |
| 2    | 0      | (0,1,0) |
| 3    |        |         |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



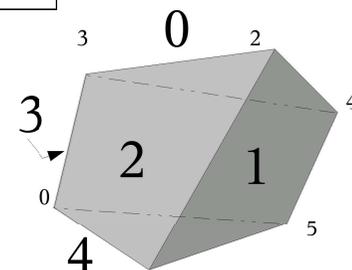
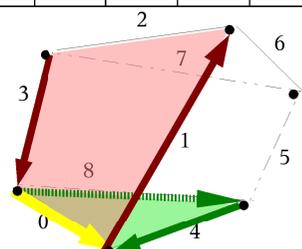
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       |    |    |    |    |     |     |     |     |
| 2       |    |    |    |    |     |     |     |     |
| 3       |    |    |    |    |     |     |     |     |
| 4       |    |    |    |    |     |     |     |     |
| 5       |    |    |    |    |     |     |     |     |
| 6       |    |    |    |    |     |     |     |     |
| 7       |    |    |    |    |     |     |     |     |
| 8       |    |    |    |    |     |     |     |     |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



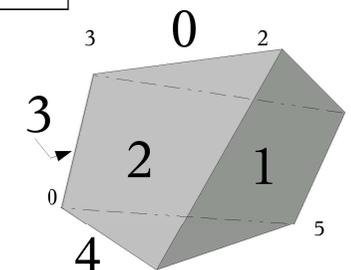
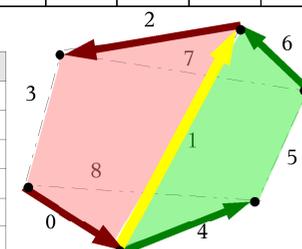
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       |    |    |    |    |     |     |     |     |
| 3       |    |    |    |    |     |     |     |     |
| 4       |    |    |    |    |     |     |     |     |
| 5       |    |    |    |    |     |     |     |     |
| 6       |    |    |    |    |     |     |     |     |
| 7       |    |    |    |    |     |     |     |     |
| 8       |    |    |    |    |     |     |     |     |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



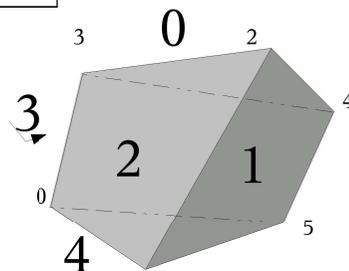
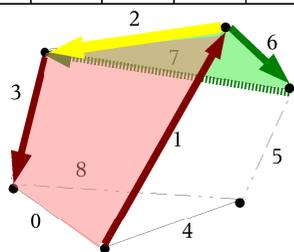
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       |    |    |    |    |     |     |     |     |
| 4       |    |    |    |    |     |     |     |     |
| 5       |    |    |    |    |     |     |     |     |
| 6       |    |    |    |    |     |     |     |     |
| 7       |    |    |    |    |     |     |     |     |
| 8       |    |    |    |    |     |     |     |     |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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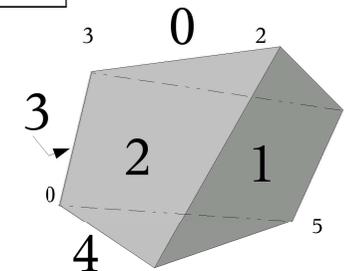
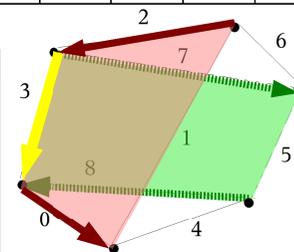
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3  | 0  | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       |    |    |    |    |     |     |     |     |
| 5       |    |    |    |    |     |     |     |     |
| 6       |    |    |    |    |     |     |     |     |
| 7       |    |    |    |    |     |     |     |     |
| 8       |    |    |    |    |     |     |     |     |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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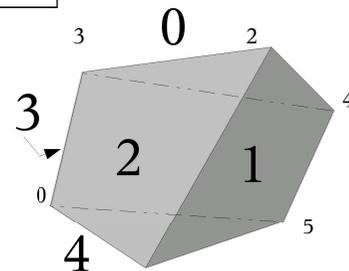
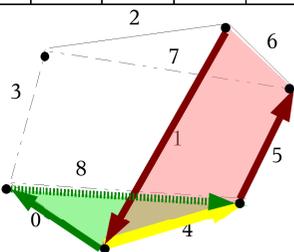
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3  | 0  | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1  | 5  | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       |    |    |    |    |     |     |     |     |
| 6       |    |    |    |    |     |     |     |     |
| 7       |    |    |    |    |     |     |     |     |
| 8       |    |    |    |    |     |     |     |     |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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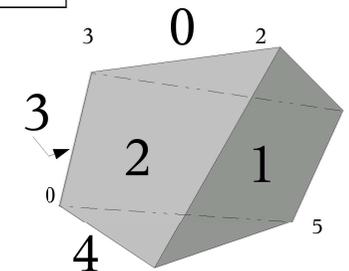
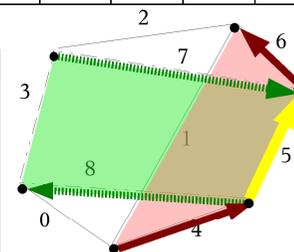
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3  | 0  | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1  | 5  | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5  | 4  | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       |    |    |    |    |     |     |     |     |
| 7       |    |    |    |    |     |     |     |     |
| 8       |    |    |    |    |     |     |     |     |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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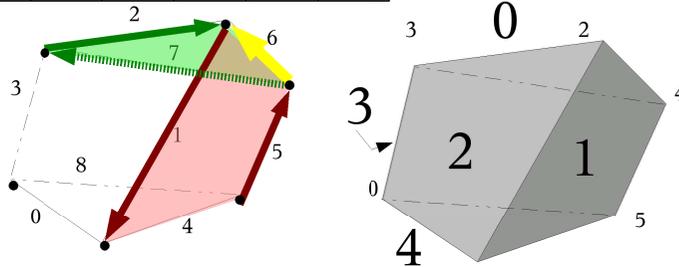
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3  | 0  | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1  | 5  | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5  | 4  | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4  | 2  | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       |    |    |    |    |     |     |     |     |
| 8       |    |    |    |    |     |     |     |     |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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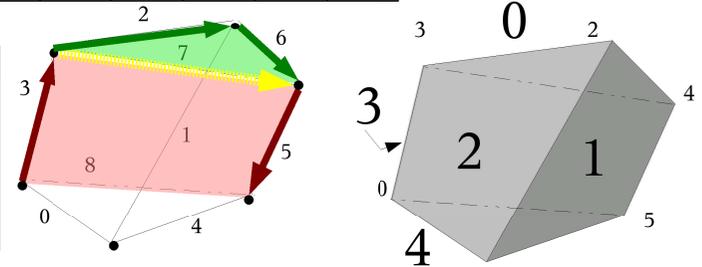
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3  | 0  | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1  | 5  | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5  | 4  | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4  | 2  | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       | 3  | 4  | 3  | 0  | 3   | 5   | 6   | 2   |
| 8       |    |    |    |    |     |     |     |     |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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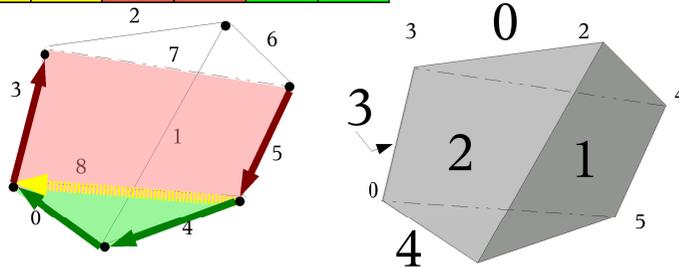
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3  | 0  | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1  | 5  | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5  | 4  | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4  | 2  | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       | 3  | 4  | 3  | 0  | 3   | 5   | 6   | 2   |
| 8       | 5  | 0  | 3  | 4  | 5   | 3   | 0   | 4   |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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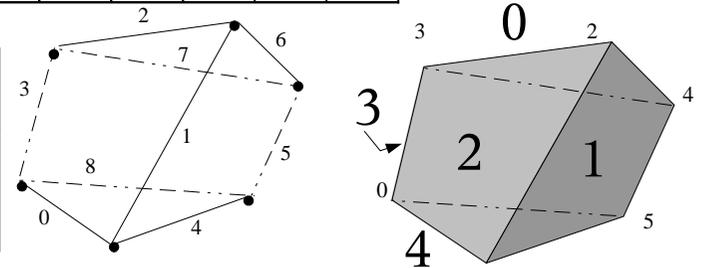
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: Creación

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3  | 0  | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1  | 5  | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5  | 4  | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4  | 2  | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       | 3  | 4  | 3  | 0  | 3   | 5   | 6   | 2   |
| 8       | 5  | 0  | 3  | 4  | 5   | 3   | 0   | 4   |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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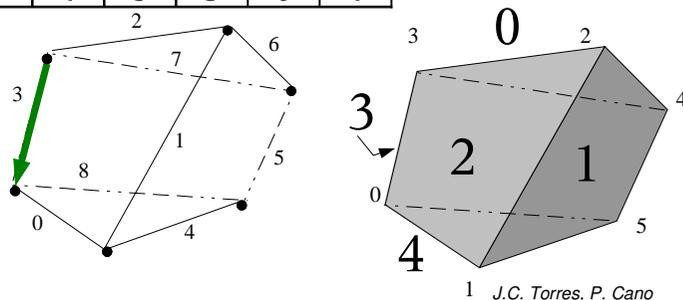
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: *Lectura de una cara*

| Aristas | Vi | Vf | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|----|----|----|----|-----|-----|-----|-----|
| 0       | 0  | 1  | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1  | 2  | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2  | 3  | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3  | 0  | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1  | 5  | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5  | 4  | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4  | 2  | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       | 3  | 4  | 3  | 0  | 3   | 5   | 6   | 2   |
| 8       | 5  | 0  | 3  | 4  | 5   | 3   | 0   | 4   |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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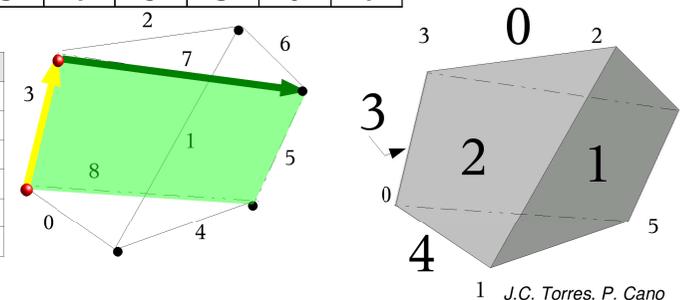
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: *Lectura de una cara*

| Aristas | Vi    | Vf    | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|-------|-------|----|----|-----|-----|-----|-----|
| 0       | 0     | 1     | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1     | 2     | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2     | 3     | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3 (2) | 0 (1) | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1     | 5     | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5     | 4     | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4     | 2     | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       | 3     | 4     | 3  | 0  | 3   | 5   | 6   | 2   |
| 8       | 5     | 0     | 3  | 4  | 5   | 3   | 0   | 4   |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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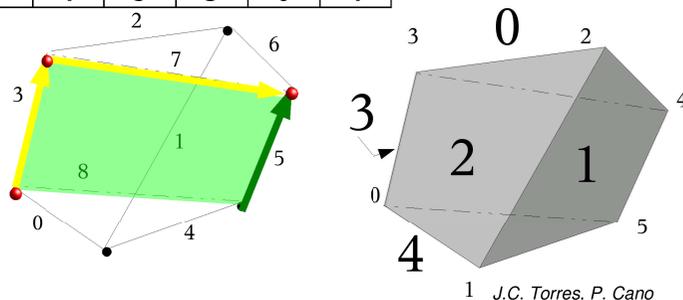
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: *Lectura de una cara*

| Aristas | Vi    | Vf    | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|-------|-------|----|----|-----|-----|-----|-----|
| 0       | 0     | 1     | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1     | 2     | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2     | 3     | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3 (2) | 0 (1) | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1     | 5     | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5     | 4     | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4     | 2     | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       | 3     | 4 (3) | 3  | 0  | 3   | 5   | 6   | 2   |
| 8       | 5     | 0     | 3  | 4  | 5   | 3   | 0   | 4   |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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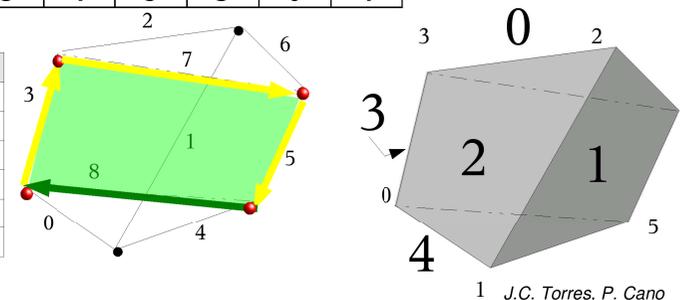
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: *Lectura de una cara*

| Aristas | Vi    | Vf    | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|-------|-------|----|----|-----|-----|-----|-----|
| 0       | 0     | 1     | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1     | 2     | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2     | 3     | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3 (2) | 0 (1) | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1     | 5     | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5 (4) | 4     | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4     | 2     | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       | 3     | 4 (3) | 3  | 0  | 3   | 5   | 6   | 2   |
| 8       | 5     | 0     | 3  | 4  | 5   | 3   | 0   | 4   |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



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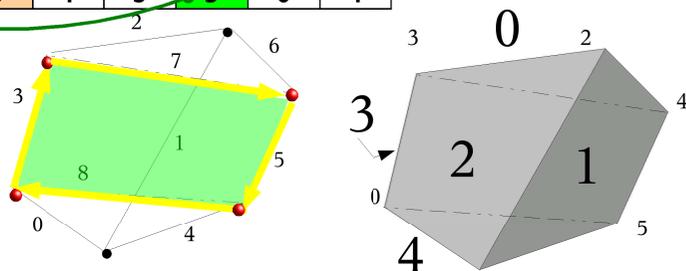
### 5.3. Modelo de fronteras: Representación

#### Aristas con alas: *Lectura de una cara*

| Aristas | Vi    | Vf    | Cd | Ci | Ada | Ads | Aia | Ais |
|---------|-------|-------|----|----|-----|-----|-----|-----|
| 0       | 0     | 1     | 2  | 4  | 3   | 1   | 4   | 8   |
| 1       | 1     | 2     | 2  | 1  | 0   | 2   | 6   | 4   |
| 2       | 2     | 3     | 2  | 0  | 1   | 3   | 7   | 6   |
| 3       | 3 (2) | 0 (1) | 2  | 3  | 2   | 0   | 8   | 7   |
| 4       | 1     | 5     | 1  | 4  | 1   | 5   | 8   | 0   |
| 5       | 5 (4) | 4     | 1  | 3  | 4   | 6   | 7   | 8   |
| 6       | 4     | 2     | 1  | 0  | 5   | 1   | 2   | 7   |
| 7       | 3     | 4 (3) | 3  | 0  | 3   | 5   | 6   | 2   |
| 8       | 5     | 0 (-) | 3  | 4  | 5   | 3   | 0   | 4   |

| Cara | Arista | Normal    |
|------|--------|-----------|
| 0    | 2      | (0,1,0)   |
| 1    | 5      | (1,0,0)   |
| 2    | 1      | (0,0,1)   |
| 3    | 3      | (-1,0,-1) |
| 4    | 4      | (0,-1,0)  |

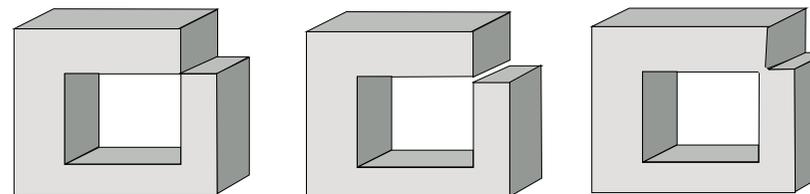
| Vértice | X | Y | Z  |
|---------|---|---|----|
| 0       | 0 | 0 | 0  |
| 1       | 0 | 2 | -1 |
| 2       | 0 | 3 | 2  |
| 3       | 0 | 1 | 2  |
| 4       | 2 | 4 | 1  |
| 5       | 2 | 3 | -1 |



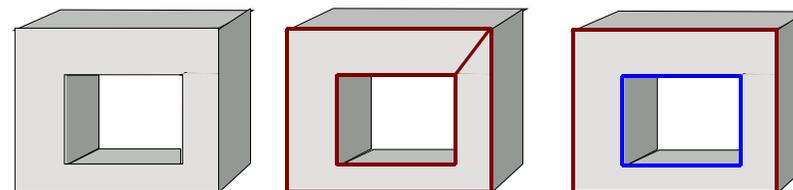
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### 5.3. Modelo de fronteras: Representación

#### Sólidos no variedad



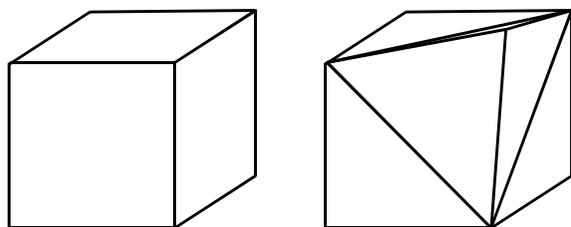
#### Sólidos con agujeros



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### 5.3. Modelo de fronteras: Edición

#### Edición directa



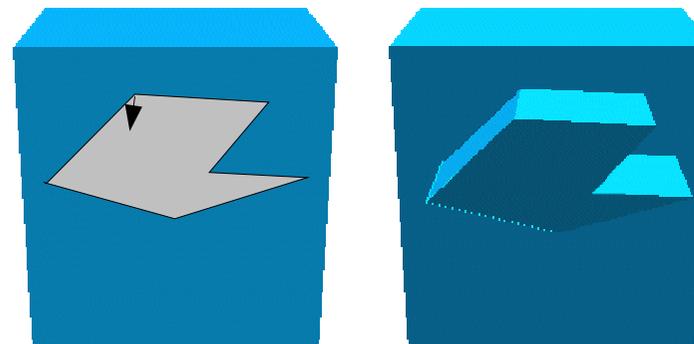
Sólo para caras triangulares

No garantiza la validez

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### 5.3. Modelo de fronteras: Edición

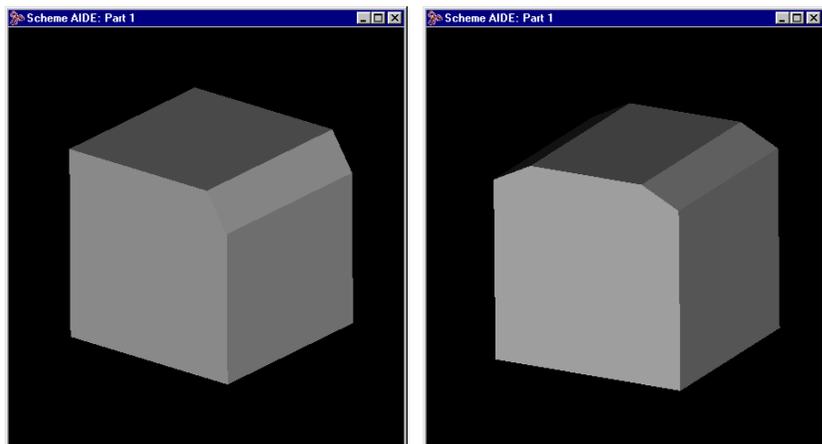
#### Barrido



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### 5.3. Modelo de fronteras: Edición

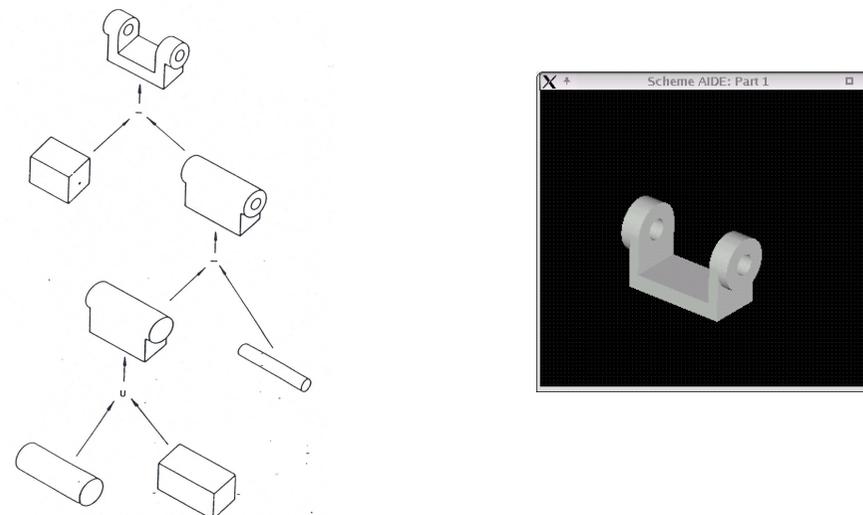
#### Biselado



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### 5.3. Modelo de fronteras: Edición

#### Operaciones booleanas



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### 5.3. Modelo de fronteras: Edición

#### Operaciones booleanas

a) Clasificación de caras

1. Calcular todas las líneas de intersección de las caras de A con las caras del sólido B y viceversa.
2. Dividir las caras que están partidas por estas líneas. Para ello se unen las aristas consecutivas entre si para formar nuevas caras.
3. Clasificar las caras de cada sólido respecto al otro sólido en externas e internas.

b) Selección de caras

4. En función de la operación a realizar se clasifican las caras eligiendo las que forman parte del sólido resultado.

| Operación  | Caras   |
|------------|---|
| $A \cup B$ | Caras de A externas a B y caras de B externas a A |
| $A \cap B$ | Caras de A internas a B y caras de B internas a A |
| $A - B$    | Caras de A externas a B y caras de B internas a A |

c) Crear representación

5. Reconstruir las relaciones entre elementos

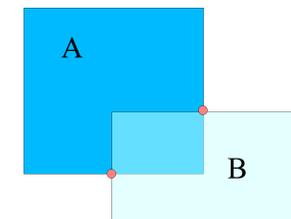
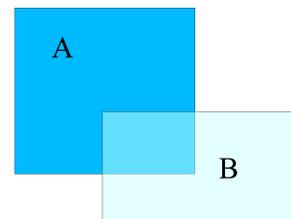
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### 5.3. Modelo de fronteras: Edición

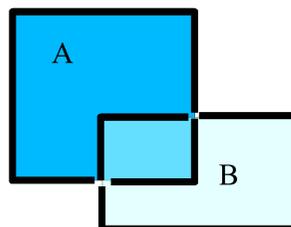
#### Operaciones booleanas

a) Clasificación de caras

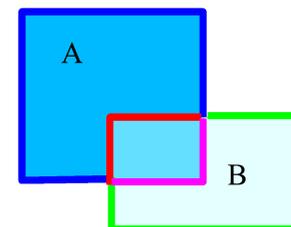
1. Calcular todas las líneas de intersección



2. Dividir las caras



3. Clasificar las caras



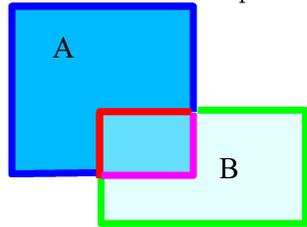
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### 5.3. Modelo de fronteras: Edición

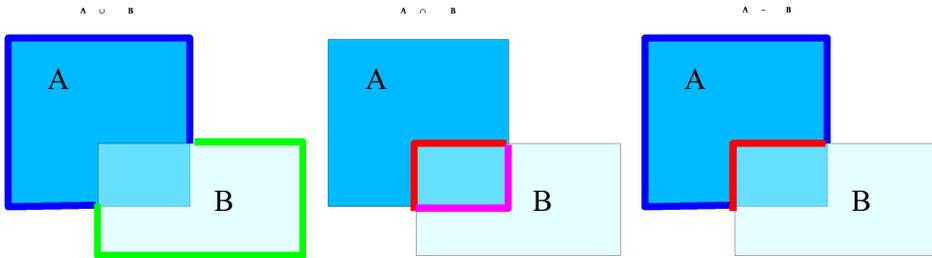
#### Operaciones booleanas

##### b) Selección de caras

4. En función de la operación se clasifican las caras



| Operación  | Caras   |
|------------|---|
| $A \cup B$ | Caras de A externas a B y caras de B externas a A |
| $A \cap B$ | Caras de A internas a B y caras de B internas a A |
| $A - B$    | Caras de A externas a B y caras de B internas a A |



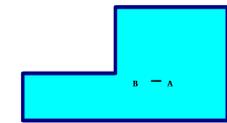
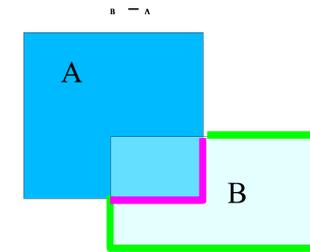
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### 5.3. Modelo de fronteras: Edición

#### Operaciones booleanas

##### c) Crear representación

5. Construir la representación

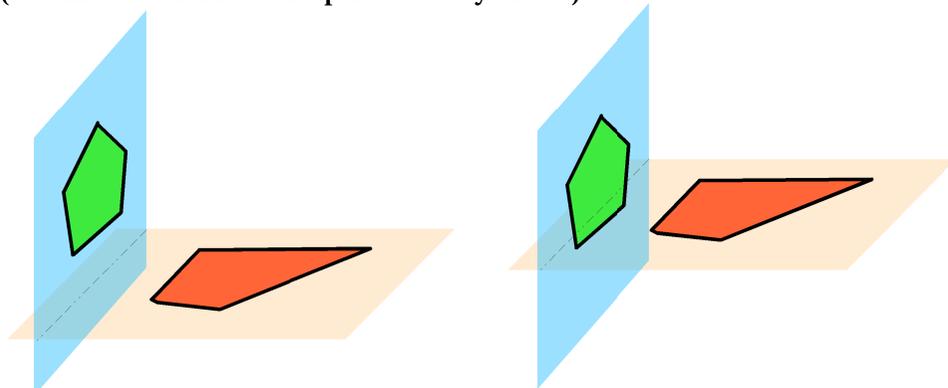


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### 5.3. Modelo de fronteras: Edición

#### Operaciones booleanas: Calculo de intersección entre caras

Si el plano de cada polígono no corta al otro polígono: no hay intersección  
(Evaluamos la ecuación del plano:  $Ax+By+Cz+D$ )

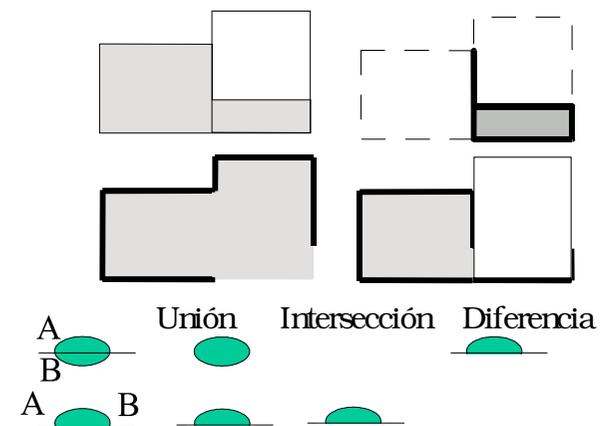


Si puede haber intersección proyectamos sobre un plano.

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### 5.3. Modelo de fronteras: Edición

#### Operaciones booleanas: Tratamiento de caras coplanarias



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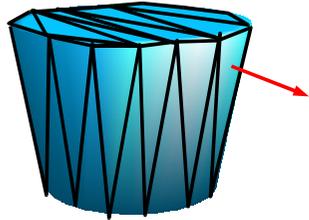
## 5.3 Modelo de fronteras: Calculo de propiedades

### Teorema de la divergencia:

$$\iiint_V (\operatorname{div} \mathbf{F}) dV = \oint_S \mathbf{F} \cdot \mathbf{n} dS$$

$$\mathbf{F}(x,y,z) = (F_x, F_y, F_z)$$

$$\operatorname{div} \mathbf{F} = \frac{\delta F_x}{\delta x} + \frac{\delta F_y}{\delta y} + \frac{\delta F_z}{\delta z}$$



Descomponemos la superficie del sólido en trozos simples con la "misma" normal y el "mismo" valor de F.

$$\oint_S \mathbf{F} \cdot \mathbf{n} dS = \sum_S \mathbf{F} \cdot \mathbf{n} dS$$

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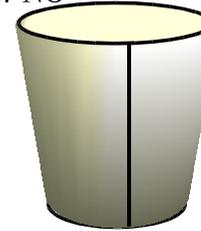
## 5.3. Modelo de fronteras: Propiedades

Dominio: Depende del tipo de caras. Poliedros si caras planas

Validez: No

Unicidad: Si (si no hay caras partidas, salvo permutación de elementos)

Ambiguo: NO



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## 5.3. Modelo de fronteras: Propiedades

### Validez

*Cualquier conjunto de caras no es un sólido!*

#### Cerrado

- i) Cada arista está delimitada por dos vértices.
- ii) Cada arista separa dos caras.

#### Orientable

- iii) Las caras que coinciden en una arista tienen orientación conforme.
- iv) Las aristas solo se intersectan en los vértices.
- v) Las caras solo se intersectan en los vértices y las aristas.

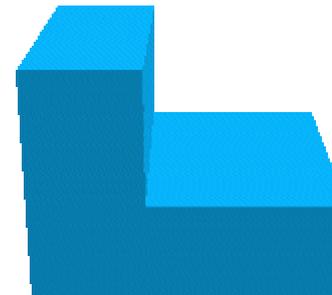
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## 5.3. Modelo de fronteras: Propiedades

### Validez: Formula de Euler

*Condición necesaria (pero no suficiente)*

caras + vértices = aristas + anillos + 2 · (cuerpos – perforaciones)



$$V + F = E + R + 2(B - H)$$

|          |    |                   |    |
|----------|----|-------------------|----|
| Vértices | 12 | Aristas           | 18 |
| + Caras  | 8  | + Anillos         | 0  |
|          | 20 | + 2 · Cuerpos     | 2  |
|          |    | - 2 · Perforación | 0  |
|          |    |                   | 20 |

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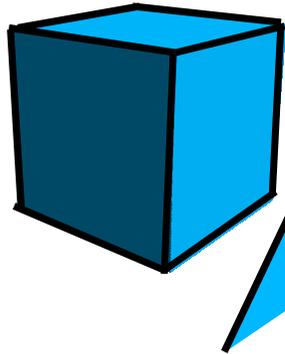
## 5.3. Modelo de fronteras: Propiedades

### Validez: Formula de Euler

*Condición necesaria (pero no suficiente)*

caras + vértices = aristas + anillos + 2 · (cuerpos – perforaciones)

$$V + F = E + R + 2 \cdot (B - H)$$



|          |     |                     |      |
|----------|-----|---------------------|------|
| Vértices | 8   | Aristas             | 12+1 |
| + Caras  | 6+1 | + Anillos           | 0    |
|          | 15  | + 2 · Cuerpos       | 2    |
|          |     | + 2 · Perforaciones | 0    |
|          |     |                     | 15   |

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## Tema 5: Modelado de sólidos

### 5.1. Introducción.

Modelos matemáticos. Propiedades volumétricas. Propiedades de los métodos de representación. Operaciones regularizadas

### 5.2 Modelos constructivos: CSG

Representación. Visualización

### 5.3. Modelo de fronteras (B-rep)

Representación. Cálculo de propiedades. Edición

### 5.4 Modelos de descomposición: Octrees

Almacenamiento. Búsqueda. Operaciones booleanas

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